Journal of Engineering Physics and Thermophysics, Vol. 74, No. 2, 2001

CALCULATION OF FREE-CONVECTIVE HEAT TRANSFER ON A VERTICAL SEMIINFINITE PLATE

V. N. Korovkin and A. P. Andrievskii UDC 536.25

Self-similar problems of free-convective heat transfer on a vertical flat semiinfinite plate for high Prandtl numbers and three types of thermal boundary conditions (an adiabatic surface, a constant temperature, and a constant heat flux on the surface) are solved by the method of internal and external expansions on the basis of the equations of a laminar boundary layer in the Boussinesq approximation. Asymptotic relations are found for the main characteristics. The results obtained are compared with the data of other authors.

The processes of free-convective heat transfer on a vertical surface with different boundary conditions, in particular, the laminar mode of motion of a liquid or a gas, have been given much consideration in the technical literature [1–4]. However, here it should be noted that, despite the theoretical and practical value of the results obtained and the numerous issues considered (the effect of the Prandtl number, variable force of gravity, energy dissipation, stratification of the environment, compressibility of a flow, etc. on the heat transfer), a number of quite significant and urgent engineering problems, associated first of all with obtaining reliable computational formulas which could allow one to predict the dynamics of the studied process depending on the main parameters, still remain to be solved completely. Use of empirical relations [4], which are characterized by a relatively high error, is ineffectual in most cases, and the results obtained based on them should be assumed to be estimative. This is due to the fact that mathematical models of free-convective heat transfer are a set of nonlinear interrelated partial differential equations, for integration of which there are no analytical methods. Therefore, these problems are solved using numerical schemes based on explicit and implicit finite-difference algorithms of calculation [5–8]. Under these conditions, investigations associated with the improvement of approximate analytical methods are of obvious interest for science and technology.

In what follows, we present results of a complex study of fully developed free-convective flows on a flat impermeable vertical semiinfinite plate for three types of thermal boundary conditions: an adiabatic surface, a constant temperature, and a constant heat flux on the surface. The analysis is based on the equations of a stationary laminar boundary layer in the Boussinesq approximation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,,\quad u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta\Delta T\,,\quad u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\nu}{\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2} \qquad (1)$$

with the corresponding boundary conditions for the velocity and temperature fields. Approximate solutions of boundary-value problems corresponding to a specific process of free-convective heat transfer on a vertical surface with the variable functions $u(x, y)$, $v(x, y)$, and $T(x, y)$ are constructed by the method of matched asymptotic expansions [9].

Isothermal Surface. In this case, the boundary conditions for Eqs. (1) are

---

Polotsk State University, Polotsk, Belarus; email: aland@psu.unibel.by. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 74, No. 2, pp. 68–72, March–April, 2001. Original article submitted August 30, 2000.

1062-0125/01/7402-0347$25.00

$$y = 0: \ \ v = u = 0 \ , \ \ T = T_w \ ; \ \ y \to \infty : \ \ u \to 0 \ , \ \ T \to T_\infty \ . \tag{2}$$

As a result of substitution of expressions of the form

$$\psi = (g\beta\Delta T_w \nu^2)^{1/4} F(\eta)\, x^{3/4} \ , \ \ \eta = \left(\frac{g\beta\Delta T_w}{\nu^2}\right)^{1/4} x^{-1/4} y \ ,$$

$$u = (g\beta\Delta T_w)^{1/2} F'(\eta)\, x^{1/2} \ , \ \ \Delta T = \Delta T_w H(\eta) \tag{3}$$

into system (1)–(2), we obtain

$$F''' + \frac{3}{4} FF'' - \frac{1}{2} F'^2 + H = 0 \ , \ \ H'' + \frac{3}{4} \Pr FH' = 0 \ ,$$

$$F(0) = 0 \ , \ \ F'(0) = 0 \ , \ \ F'(\infty) = 0 \ , \ \ H(0) = 1 \ , \ \ H(\infty) = 0 \ . \tag{4}$$

The following step is designed to simplify the analysis of the problem further by "splitting" a mathematical model into submodels, i.e., the equations and conditions (4) which formulate the laws governing the studied process are "split" into two systems of equations. In this case, the procedure of "splitting" is associated with the physical features of free convective heat transfer on a vertical isothermal surface: when $\Pr > 1$, heat transfer occurs in a thin thermal boundary layer of thickness $\delta_t \sim \Pr^{-1/4}$. Outside the thermal boundary layer, there exists a viscous boundary layer of thickness $\delta_v \sim \Pr^{1/4}$ ($\delta_v/\delta_t = \Pr^{1/2}$) where the flow does not depend on buoyancy forces (H) and occurs due to the entrainment of a liquid from the environment by friction. Following [10–12], we introduce the internal and external variables below

$$F(\eta) = \Pr^{-3/4} f(\zeta) \ , \ \ \zeta = \Pr^{1/4} \eta \ , \ \ H(\eta) = h(\zeta) \ ;$$

$$F(\eta) = \gamma \Pr^{-1/4} G(z) \ , \ \ z = \gamma \Pr^{-1/4} \eta \ , \ \ H(\eta) = 0 \ , \tag{5}$$

and represent the functions $f(\zeta)$, $h(\zeta)$, and $G(z)$ in the form of infinite series in powers of a small parameter $\varepsilon = \Pr^{-1/2}$. Substituting (3) into the system of equations (4) and collecting the terms with the same powers ε, we obtain equations for determining the unknown functions f_i, h_i, and G_i:

the internal problem

$$f_0''' + h_0 = 0 \ , \ \ h_0'' + \frac{3}{4} f_0 h_0' = 0 \ , \ \ f_0(0) = 0 \ , \ \ f_0'(0) = 0 \ , \ \ h_0(0) = 1 \ ,$$

$$f_i''' + h_i = -\frac{3}{4} \sum_{j=1}^{i-1} f_{j-1} f_{i-j-1}'' + \frac{1}{2} \sum_{j=1}^{i-1} f_{j-1}' f_{i-j-1}' \ , \tag{6}$$

$$h_i'' + \frac{3}{4} f_0 h_i' = -\frac{3}{4} \sum_{j=1}^{i} f_j h_{i-j}' \ , \ \ f_i(0) = 0 \ , \ \ f_i'(0) = 0 \ , \ \ h_i(0) = 0 \ ;$$

the external problem

$$G_0''' + \frac{3}{4} G_0 G_0'' - \frac{1}{2} G_0'^2 = 0 \ ,$$

$$G_i''' + \frac{3}{4} G_0 G_i'' - G_0' G_i' + \frac{3}{4} G_0'' G_i = -\frac{3}{4} \sum_{j=1}^{i-1} G_j G_{i-j}'' + \frac{1}{2} \sum_{j=1}^{i-1} G_j' G_{i-j}' , \quad G_i' (\infty) = 0 , \tag{7}$$

where the prime denotes differentiation with respect to the corresponding variable ζ or z. The boundary conditions at $\zeta = \infty$ and $z = 0$ are found from the condition of matching of the internal and external expansions:

$$\lim_{\zeta \to \infty} \left(f_0 + \mathrm{Pr}^{-1/2} f_1 + \mathrm{Pr}^{-1} f_2 + ... \right) = \gamma \, \mathrm{Pr}^{1/2} \lim_{z \to 0} \left(G_0 + \mathrm{Pr}^{-1/2} G_1 + \mathrm{Pr}^{-1} G_2 + ... \right),$$

$$\lim_{\zeta \to \infty} \left(h_0 + \mathrm{Pr}^{-1/2} h_1 + \mathrm{Pr}^{-1} h_2 + ... \right) = 0 . \tag{8}$$

Constant Heat Flux on the Surface. The solution of system (1) must satisfy the boundary conditions

$$y = 0 : \quad u = v = 0 , \quad -k \frac{\partial T}{\partial y} = q_{\mathrm{w}} = \mathrm{const} ; \quad y \to \infty : \quad u \to 0 , \quad T \to T_{\infty} . \tag{9}$$

Allowing for (9), we introduce new variables

$$\psi = \left(\frac{g \beta q_{\mathrm{w}} \nu^3}{k} \right)^{1/5} F (\eta) \, x^{4/5} , \quad \eta = \left(\frac{g \beta q_{\mathrm{w}}}{k \nu^2} \right)^{1/5} x^{-1/5} y ,$$

$$u = \left(\frac{g \beta q_{\mathrm{w}} \nu^{1/2}}{k} \right)^{2/5} F' (\eta) \, x^{3/5} , \quad \Delta T = \frac{q_{\mathrm{w}}}{k} \left(\frac{k \nu^2}{g \beta q_{\mathrm{w}}} \right)^{1/5} H (\eta) \, x^{1/5} \tag{10}$$

and rewrite the system of equations (1) and (9) in the form

$$F''' + \frac{4}{5} F F'' - \frac{3}{5} F'^2 + H = 0 , \quad H'' + \mathrm{Pr} \left(\frac{4}{5} F H' - \frac{1}{5} F' H \right) = 0 ,$$

$$F (0) = 0 , \quad F' (0) = 0 , \quad F' (\infty) = 0 , \quad H' (0) = -1 , \quad H (\infty) = 0 . \tag{11}$$

Then, since in the case of boundary conditions of the second kind at large Prandtl numbers $\delta_{\mathrm{t}} \sim \mathrm{Pr}^{-1/5}$ and $\delta_{\mathrm{v}} \sim \mathrm{Pr}^{-3/10}$, we represent, similarly to [13, 14], the sought functions $F(\eta)$ and $H(\eta)$ for the internal and external layers, respectively, as

$$F (\eta) = \mathrm{Pr}^{-4/5} f (\zeta) , \quad \zeta = \mathrm{Pr}^{1/5} \eta , \quad H (\eta) = \mathrm{Pr}^{-1/5} h (\zeta) ;$$

$$F (\eta) = \gamma \, \mathrm{Pr}^{-3/10} G (z) , \quad z = \gamma \, \mathrm{Pr}^{-3/10} \eta , \quad H (\eta) = 0 . \tag{12}$$

As a result, we come to the necessity of integrating the following chain of interrelated ordinary differential equations:

the internal problem

$$f_0''' + h_0 = 0 , \quad h_0'' + \frac{4}{5} f_0 h_0' - \frac{1}{5} f_0' h_0 = 0 , \quad f_0 (0) = 0 , \quad f_0' (0) = 0 , \quad h_0' (0) = -1 ,$$

$$f_i''' + h_i = -\frac{4}{5}\sum_{j=1}^{i-1} f_{j-1} f''_{i-j-1} + \frac{3}{5}\sum_{j=1}^{i-1} f'_{j-1} f'_{i-j-1} ,$$

$$h''_i + \frac{4}{5} f_0 h'_i - \frac{1}{5} f'_0 h_i = -\frac{4}{5}\sum_{j=1}^{i} f_j h'_{i-j} + \frac{1}{5}\sum_{j=1}^{i} f'_j h_{i-j} , \quad f_i(0) = 0 , \quad f'_i(0) = 0 , \quad h'_i(0) = 0 ; \tag{13}$$

the external problem

$$G_0''' + \frac{4}{5} G_0 G''_0 - \frac{3}{5} G_0'^2 = 0 , \quad G_i''' + \frac{4}{5} G_0 G''_i - \frac{6}{5} G'_0 G'_i + \frac{4}{5} G''_0 G_i = -\frac{4}{5}\sum_{j=1}^{i-1} G_j G''_{i-j} + \frac{3}{5}\sum_{j=1}^{i-1} G'_j G'_{i-j} ,$$

$$G'_i(\infty) = 0 . \tag{14}$$

By virtue of (8), solutions of each preceding system of equations are the boundary conditions for the subsequent system. Thus, using the field of the unknown functions, obtained in integration of the preceding equations, as the boundary relations at $\zeta = \infty$ and $z = 0$, we perform step-by-step "joining" of solutions. The special properties of Eqs. (13) and (14), in contrast to (11), are that those equations do not contain the parameter Pr; therefore the functions $f_i(\zeta)$, $h_i(\zeta)$, and $G_i(z)$ will not depend on Pr.

Adiabatic Surface. In this case, the boundary conditions are written as

$$y = 0 : \; u = v = 0 , \; \frac{\partial T}{\partial y} = 0 ; \; y \to \infty : \; u \to 0 , \; T \to T_\infty . \tag{15}$$

If we pass to the self-similar variables

$$\psi = \left(\frac{g\beta Q_0 \nu^2}{\rho C_p}\right)^{1/5} F(\eta)\, x^{3/5} , \quad \eta = \left(\frac{g\beta Q_0}{\rho C_p \nu^3}\right)^{1/5} x^{-2/5} y ,$$

$$u = \left(\frac{g\beta Q_0}{\rho C_p \nu^{1/2}}\right)^{2/5} F'(\eta)\, x^{1/5} , \quad \Delta T = \left(\frac{Q_0}{(g\beta)^{1/4} \rho C_p \nu^{1/2}}\right)^{4/5} H(\eta)\, x^{-3/5} , \tag{16}$$

where

$$Q_0 = \rho C_p \int_0^\infty u \Delta T dy = \text{const} ,$$

then determination of the unknown functions $F(\eta)$ and $H(\eta)$ is reduced to solution of the following boundary-value problem:

$$F''' + \frac{3}{5} F F'' - \frac{1}{5} F'^2 + H = 0 , \quad H'' + \text{Pr}\left(\frac{3}{5} F H' + \frac{3}{5} F' H\right) = 0 ,$$

$$F(0) = 0 , \quad F'(0) = 0 , \quad F'(\infty) = 0 , \quad H'(0) = 0 , \quad H(\infty) = 0 , \quad \int_0^\infty F' H \, d\eta = 1 . \tag{17}$$

At large values of Pr the boundary layer on the vertical adiabatic surface can be subdivided into two regions: one of thickness $\delta_t \sim \mathrm{Pr}^{-2/5}$, where the temperature difference tends to zero and the other of thickness $\delta_v \sim \mathrm{Pr}^{1/10}$, where the velocity $u \to 0$. Proceeding from the above, we introduce the following variables and functions [15] for the internal and external layers, respectively:

$$F(\eta) = \mathrm{Pr}^{-3/5} f(\zeta)\,, \quad \zeta = \mathrm{Pr}^{2/5}\eta\,, \quad H(\eta) = \mathrm{Pr}^{3/5} h(\zeta)\,;$$
$$F(\eta) = \gamma\,\mathrm{Pr}^{-1/10} G(z)\,, \quad z = \gamma\,\mathrm{Pr}^{-1/10}\eta\,, \quad H(\eta) = 0\,. \tag{18}$$

By virtue of expressions (18) and the representation of the functions $f(\zeta)$, $h(\zeta)$, and $G(z)$ by expansions into power series ε, the system of equations (17) is rewritten as follows:

the internal problem

$$f_0''' + h_0 = 0\,, \quad h_0'' + \frac{3}{5} f_0 h_0' + \frac{3}{5} f_0' h_0 = 0\,, \quad \int_0^\infty f_0' h_0\, d\zeta = 1\,, \quad f_0(0) = 0\,, \quad f_0'(0) = 0\,, \quad h_0'(0) = 0\,,$$

$$f_i''' + h_i = -\frac{3}{5}\sum_{j=1}^{i-1} f_{j-1} f_{i-j-1}'' + \frac{1}{5}\sum_{j=1}^{i-1} f_{j-1}' f_{i-j-1}'\,, \quad h_i'' + \frac{3}{5} f_0 h_i' + \frac{3}{5} f_0' h_i = -\frac{3}{5}\sum_{j=1}^{i} f_j h_{i-j}' - \frac{3}{5}\sum_{j=1}^{i} f_j' h_{i-j}\,, \tag{19}$$

$$\int_0^\infty \left(f_0' h_i + \sum_{j=1}^{i} f_j' h_{i-j} \right) d\zeta = 0\,, \quad f_i(0) = 0\,, \quad f_i'(0) = 0\,, \quad h_i'(0) = 0\,;$$

the external problem

$$G_0''' + \frac{3}{5} G_0 G_0'' - \frac{1}{5} G_0'^2 = 0\,,$$
$$G_i''' + \frac{3}{5} G_0 G_i'' - \frac{2}{5} G_0' G_i' + \frac{3}{5} G_0'' G_i = -\frac{3}{5}\sum_{j=1}^{i-1} G_j G_{i-j}'' + \frac{1}{5}\sum_{j=1}^{i-1} G_j' G_{i-j}'\,, \quad G_i'(\infty) = 0\,. \tag{20}$$

Calculation Results and Their Analysis. To complete the formulation of the problem, we must write the conditions of matching of the internal and external asymptotic expansions. For the zeroth approximation we write, in accordance with (8),

$$f_0''(\infty) = 0\,, \quad h_0(\infty) = 0\,, \quad G_0(0) = 0\,, \quad G_0'(0) = \frac{f_0'(\infty)}{\gamma^2} = 1 \quad (\gamma^2 = f_0'(\infty))\,. \tag{21}$$

Similarly, we can find additional relations for higher-order approximations. Thus, having found the functions $f_i(\zeta)$, $h_i(\zeta)$, and $G_i(z)$, we can construct, by calculations, the profiles of velocity (temperature) at different Pr numbers and calculate local Nusselt numbers, friction stress on the plate, and mass flow rate of the liquid per second in the boundary layer:

an isothermal surface ($\mathrm{Gr}_x = g\beta\Delta T_w x^3/\nu^2$)

$$\mathrm{Nu}_x\,\mathrm{Gr}_x^{-1/4} = -\,\mathrm{Pr}^{1/4}\sum_{i=0}^{\infty} h_i'(0)\,\varepsilon^i\,, \quad \frac{\tau_w x^2}{\rho\nu^2}\,\mathrm{Gr}_x^{-3/4} = \mathrm{Pr}^{-1/4}\sum_{i=0}^{\infty} f_i''(0)\,\varepsilon^i\,, \quad \frac{m}{\mu}\,\mathrm{Gr}_x^{-1/4} = \mathrm{Pr}^{-1/4}\gamma\sum_{i=0}^{\infty} G_i(\infty)\,\varepsilon^i\,;$$

TABLE 1. Asymptotic Characteristics of Free-Convective Heat Transfer on a Vertical Impermeable Semiinfinite Flat Surface

Thermal boundary conditions	$-h_0'(0)$	$h_0(0)$	$f_0''(0)$	$\gamma G_0(\infty)$	Reference
Isothermal surface	0.5027451	1	1.1660423	–	[10]
	0.50274	1	1.16597	–	[12]
	0.5027454	1	1.1660422	1.2139858	Our data
Constant heat flux on the surface	1	1.58320	1.59505	–	[13]
	1	1.583329	1.544903	–	[14]
	1	1.5840	–	–	[16]
	1	1.5831587	1.5454761	1.2400671	Our data
Adiabatic surface	0	1.58145	1.18035	1.65591	[15]
	0	0.5814385	1.1803498	1.6559780	Our data

a constant heat flux $\mathrm{Gr}_x^* = g\beta q_w x^4/(k\nu^2)$ is specified on the surface

$$\mathrm{Nu}_x (\mathrm{Gr}_x^*)^{-1/5} = \frac{\mathrm{Pr}^{1/5}}{\sum_{i=0}^{\infty} h_i(0)\,\varepsilon^i}\,, \quad \frac{\tau_w x^2}{\rho\nu^2}(\mathrm{Gr}_x^*)^{-3/5} = \mathrm{Pr}^{-2/5}\sum_{i=0}^{\infty} f_i''(0)\,\varepsilon^i\,, \quad \frac{m}{\mu}(\mathrm{Gr}_x^*)^{-1/5} = \mathrm{Pr}^{-3/10}\gamma\sum_{i=0}^{\infty} G_i(\infty)\,\varepsilon^i\,; \tag{22}$$

an adiabatic surface ($\mathrm{Gr}_x = g\beta Q_0 x^3/(\rho C_p \nu^3)$)

$$\frac{\Delta T_w\,\mu C_p}{Q_0}\mathrm{Gr}_x^{1/5} = \mathrm{Pr}^{3/5}\sum_{i=0}^{\infty} h_i(0)\,\varepsilon^i\,, \quad \frac{\tau_w x^2}{\rho\nu^2}\mathrm{Gr}_x^{-3/5} = \mathrm{Pr}^{1/5}\sum_{i=0}^{\infty} f_i''(0)\,\varepsilon^i\,, \quad \frac{m}{\mu}\mathrm{Gr}_x^{-1/5} = \mathrm{Pr}^{-1/10}\gamma\sum_{i=0}^{\infty} G_i(\infty)\,\varepsilon^i\,.$$

The functions f_0, h_0, and G_0 were found numerically by the Runge–Kutta–Merson method by reducing (6)–(7), (13)–(14), and (19)–(20) to the corresponding Cauchy problems. As is shown by the studies of [17], in numerical integration of such equations there arise certain difficulties which are associated with the strong dependence of the behavior of the sought functions on missing boundary conditions at $\zeta = 0$: converging solutions exist only within a very narrow range of arbitrarily specified quantities. This behavior of the system of equations is known in computational mathematics as a "rigid problem." Therefore, in the numerical calculation it is desirable to have a technique which allows one to overcome the indicated difficulties. In this work, this is done by finding the initial parameters in advance. The idea of this approach is to split the functions f_0 and h_0 into n terms, with the solution corresponding to the case where $f_0''' = 0$ is taken as the first approximation. Then we construct two (three) terms of the series, using which we find formulas for determining the estimative values of $f_0''(0)$ and $h_0(0)$ (or $h_0'(0)$). The last procedure corresponds to summation of a certain infinite number subsequence which enters into the main series. Similarily, we can write the expression for $\gamma G_0(\infty)$ in explicit form. We note that this technique was tested in solving applied problems of jet hydrodynamics and proved to be reliable and efficient [18].

As a result, we obtained the following values (I for an isothermal surface, II for a constant heat flux specified on the surface, and III for an adiabatic surface):

$$\text{I.}\quad f_0''(0)=\left(\frac{10181}{558\pi^2}\right)^{1/4},\quad -h_0'(0)=\left(\frac{186}{295\pi^2}\right)^{1/4},\quad \gamma G_0(\infty)=\left(\frac{2701}{126\pi^2}\right)^{1/4};$$

$$\text{II.}\quad f_0''(0)=\left(\frac{11105}{72\pi^2\sqrt{\pi}}\right)^{1/5},\quad h_0(0)=\left(\frac{16528}{95\pi^2\sqrt{\pi}}\right)^{1/5},\quad \gamma G_0(\infty)=\left(\frac{18853}{2000\pi^{4/3}}\right)^{3/10};\qquad (23)$$

$$\text{III.}\quad f_0''(0)=\left(\frac{1002}{25\pi^2\sqrt{\pi}}\right)^{1/5},\quad h_0(0)=\left(\frac{93}{80\pi^2\sqrt{\pi}}\right)^{1/5},\quad \gamma G_0(\infty)=\left(\frac{618}{25\pi^{4/3}}\right)^{3/10}.$$

The data of the numerical integration are given in Table 1. The same table presents the results found earlier [10, 12–16]. It appeared that the quantities calculated using the numerical scheme and relations (23) virtually coincide: the maximum absolute error in calculating by formulas (23) is $\sim 4{\cdot}10^{-7}$. This indicates that the potentialities of analytical approaches to the study of problems of free-convective heat transfer, despite the constantly progressing development of computers and computational mathematics, are far from being exhausted.

As for the accuracy of the constructed asymptotic solutions, if we take a 5% difference in the results of calculation of, for example, the mass flow rate of a liquid per second in the boundary layer as the criterion, we can say that agreement with the data of the numerical solution of Eqs. (4), (11), and (17) [1, 4, 19] begins at Pr $\approx$10 for the boundary conditions (9) and (15) and at Pr $\approx$ 5 for relations (2). The latter indicates that for the problems considered in the present work, obtaining two (three) approximations based on the theory of matched asymptotic expansions, which is equivalent to retention of the first two (three) terms of the series in (22), makes it possible to develop a simple and practically convenient mathematical apparatus for studying the characteristic features and laws governing free-convective heat transfer. Results found within the framework of asymptotic models should be considered as a substantial addition to the information obtained on the basis of numerical integration. The method of matched asymptotic expansions has a number of advantages over the numerical techniques of solution of the boundary-value problems (4), (11), and (17). First, the method "copes" better with the description of hydrodynamics and heat transfer at large Pr numbers, and second, the algorithmic structure of the method is substantially simpler than the corresponding structures of numerical schemes, which leads to significant savings in the time needed to obtain the desired information. And finally, the fundamental possibility of determining rather accurately the character of the behavior of the solutions of the studied problem in advance with variation of the operating parameters allows one to develop a strategy of numerical analysis which provides a significant decrease in the total number of iterative solutions.

NOTATION

u and v, longitudinal and transverse components of the velocity; x and y, longitudinal and transverse coordinates; T, temperature; T_w and T_∞, temperatures of the wall and the environment; q, heat flux; k, thermal conductivity; ν, kinematic viscosity; μ, dynamic viscosity; $\Delta T = T - T_\infty$, excess temperature; ρ, density; C_p, heat capacity at constant pressure; β, coefficient of volumetric thermal expansion; m, mass flow rate per second; τ_w, friction stress on the wall; δ_t and δ_v, thickness of the thermal and viscous boundary layers; γ, normalization factor; Gr_x and Nu_x, local Grashof and Nusselt numbers; Pr, Prandtl number; ψ, stream function. Subscripts: w, wall; t, thermal; v, viscous.

REFERENCES

1. O. G. Martynenko and Yu. A. Sokovishin, *Free-Convective Heat Transfer on a Vertical Surface (Boundary Conditions of the IInd Kind)* [in Russian], Minsk (1977).

2. Yu. A. Sokovishin and O. G. Martynenko, *Introduction to the Theory of Free-Convective Heat Transfer* [in Russian], Leningrad (1982).
3. Y. Jaluria, *Natural Convection* [Russian translation], Moscow (1983).
4. B. Gebhart, Y. Jaluria, R. L. Mahajan, and B. Sammakia, *Buoyancy-Induced Flows and Transport*, Washington (1988).
5. S. V. Patankar and D. B. Spalding, *Heat and Mass Transfer in Boundary Layers*, Morgan-Grampian, London (1967).
6. V. M. Paskonov, V. I. Polezhaev, and L. A. Chudov, *Numerical Simulation of Heat and Mass Transfer Processes* [in Russian], Moscow (1984).
7. T. M. Shih, *Numerical Heat Transfer*, Washington (1984).
8. T. Cebeci and P. Bradshaw, *Physics and Computational Aspects of Convective Heat Transfer*, Springer-Verlag, New York–Berlin–Heidelberg–Tokyo (1984).
9. O. G. Martynenko, A. A. Berezovskii, and Yu. A. Sokovishin, *Asymptotic Methods in the Theory of Free-Convective Heat Transfer* [in Russian], Minsk (1979).
10. E. J. Le Ferve, in: *Proc. Int. Congr. of Appl. Mech.*, Vol. 4 (1956), pp. 168–173.
11. H. K. Kuiken, *J. Eng. Math.*, **2**, 355–371 (1968).
12. S. Roy, *Int. J. Heat Mass Transfer*, **12**, 239–241 (1969).
13. J. R. Selman and J. Newman, *Trans. ASME, Ser. C, J. Heat Transfer* [Russian translation], No. 4, 143–144 (1971).
14. S. Roy, *Trans. ASME, Ser. C, J. Heat Transfer* [Russian translation], No. 1. 135–137 (1973).
15. Y. Joshi, *Int. J. Heat Mass Transfer*, **30**, 2686–2690 (1987).
16. T. Fujii and M. Fujii, *Int. J. Heat Mass Transfer*, **19**, 121–122 (1976).
17. V. N. Korovkin and A. P. Andrievskii, *Inzh.-Fiz. Zh.*, **73**, No. 2, 381–386 (2000).
18. O. G. Martynenko, V. N. Korovkin, and Yu. A. Sokovishin, *Theory of Buoyancy Jets and Wakes* [in Russian], Minsk (1991).
19. V. N. Korovkin and A. P. Andrieevskii, *Inzh.-Fiz. Zh.*, **70**, No. 1, 142–145 (1997).